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The Berry phase in GaAs semiconductor with a quantized field

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Abstract

In this paper we investigate the Berry phase in GaAs semiconductor with a quantized magnetic field in the rotating wave approximation. The eigenfunctions of the nuclear spin in the quantized external field are obtained and thus the Berry phase is evaluated explicitly in terms of the introduction of the phase shift. It is shown that the Berry phase can be easily controlled by the coupling strength, the anisotropy constant and the frequency of the electromagnetic wave, which can be important in applications in geometric quantum computing.

(Some figures in this article are in colour only in the electronic version)

GaAs semiconductor has been regarded as an important solid-state system for processing quantum information and implementing quantum computing [1] since experiments have shown that electron spins in GaAs semiconductor can preserve their coherence for distances of more than 100 μm and for times up to 130 ns [2, 3], which is due to the weak interaction between the nuclear spins and the environment. Furthermore, the coherent control of electron and nuclear spins based on the hyperfine interaction between electrons and nuclei is experimentally accessible by means of the optical nuclear magnetic resonance technique [4–6]. Such a control of nuclear spins can also be achieved via electrical gates as investigated for GaAs heterostructures in the quantum Hall regime [7]. It is also interesting that in the presence of the nuclear quadrupole interaction, Grover's algorithm has been implemented [8, 9]. It should be noticed that above investigations are limited to the framework of the classical controlled external field, namely, this external field itself has never been quantized. It is known in quantum optics that a quantized field can lead to many novel quantum effects such as quantum jumps, collapses and revivals of the Rabi oscillations. Moreover, if the quantum system interacts with the vacuum, spontaneous emission and Lamb shift can also be observed in experiment. Latterly, a novel Berry phase, which has no zero value in the vacuum state, can also be induced if the

quantized field is controlled adiabatically and periodically [10, 11]. However, this investigation so far has been restricted to the spin-1/2 Jaynes–Cummings model. Here we extend this method to GaAs semiconductor for the spin-3/2 case. In contrast to the previous result, the Berry phase obtained by the present paper can be controlled by the matter–field coupling strength, the anisotropy constant and the frequency of the electromagnetic wave, which may be important for use in geometric quantum computing [12–17].

The Berry phase [18], which is an important topic in modern physics, describes a novel phase factor of the wavefunctions depending only on the geometry of the path when a time-dependent quantum system undergoes an adiabatic and cyclic evolution. With the exception of the discussions of the relaxation of its restriction conditions [19–22] the Berry phase has been extensively generalized in various directions, such as Berry phases for mixed states [23], for open systems [24], for composite systems [25] and for general quantum states [26], etc. Recently, the Berry phase has been regarded as an essential way to implement operation of a universal quantum logic gate in quantum computing [12–17] and as an important tool for detecting quantum phase transitions [27–30]. In condensed matter physics a series of phenomena have been understood as a manifestation of topological or geometric phases [31–36].

In the rotating wave approximation the Hamiltonian of nuclear spin in the GaAs semiconductor with a quantized magnetic field can be given by [37]

$$H = A(3S_z^2 - S^2) - \omega S_z + \omega_0 a^\dagger a - \lambda(aS_+ + a^\dagger S_-), \tag{1}$$

where λ measures the coupling strength of the nuclear spin and the quantum field, the frequency ω describes the nuclear level splitting, $a^\dagger$ and a are the photon creation and annihilation operators with the frequency ω_0, and A is the anisotropy constant differing significantly among the various nuclei. The all-optical nuclear magnetic resonance method yields the following anisotropy constant for Ga and As nuclei in GaAs semiconductors: $A = 7 \times 10^{-7}$ K for ^{69}Ga, $A = 3 \times 10^{-7}$ K for ^{71}Ga and $A = 2 \times 10^{-6}$ K for ^{75}Ga [5, 6]. S denotes the total spin operator. For GaAs semiconductor the experimentally feasible spin quantum number is $S = 3/2$ [7]. The spin operators S_z, S_+ and S_- satisfy $SU(2)$ commutation relations defined as

$$[S_z, S_\pm] = \pm S_\pm, \qquad [S_+, S_-] = 2S_z, \tag{2}$$

where $S_\pm = (S_x \pm \mathrm{i}S_y)$. In the computation vectors of the Hilbert space such that $\{|\frac{3}{2}, n\rangle, |\frac{1}{2}, n+1\rangle, |-\frac{1}{2}, n+2\rangle, |-\frac{3}{2}, n+3\rangle\}$, the matrix of Hamiltonian (1) is given by

$$H = \begin{pmatrix} 3A - \frac{3\omega}{2} + n\omega_0 & -\lambda\sqrt{3(n+1)} & 0 & 0 \\ -\lambda\sqrt{3(n+1)} & -3A - \frac{\omega}{2} + (n+1)\omega_0 & -2\lambda\sqrt{(n+2)} & 0 \\ 0 & -2\lambda\sqrt{(n+2)} & -3A + \frac{\omega}{2} + (n+2)\omega_0 & -\lambda\sqrt{3(n+3)} \\ 0 & 0 & -\lambda\sqrt{3(n+3)} & 3A + \frac{3\omega}{2} + (n+3)\omega_0 \end{pmatrix}. \tag{3}$$

The instantaneous eigenvalues E_j ($j = 1, 2, 3, 4$) of Hamiltonian (3) can be found analytically; however, it is pointless to display the tedious formula in the present paper. The ground-state energy as a function of the coupling strength λ and the anisotropy constant A in the resonance case with $\omega = \omega_0$ is shown in figure 1(a), from which we can see that both the coupling strength λ and the anisotropy constant A can control the ground-state energy as well as the Berry phase as will be shown. Figure 1(b) shows the ground-state energy as a function of the coupling strength λ and the frequency ω_0, which can be easily changed by adjusting the external quantized field. The ground-state energy as a function of the anisotropy constant A and the frequency ω_0 is plotted in figure 1(c). The eigenstates $|\psi_j\rangle$ corresponding to the eigenvalues

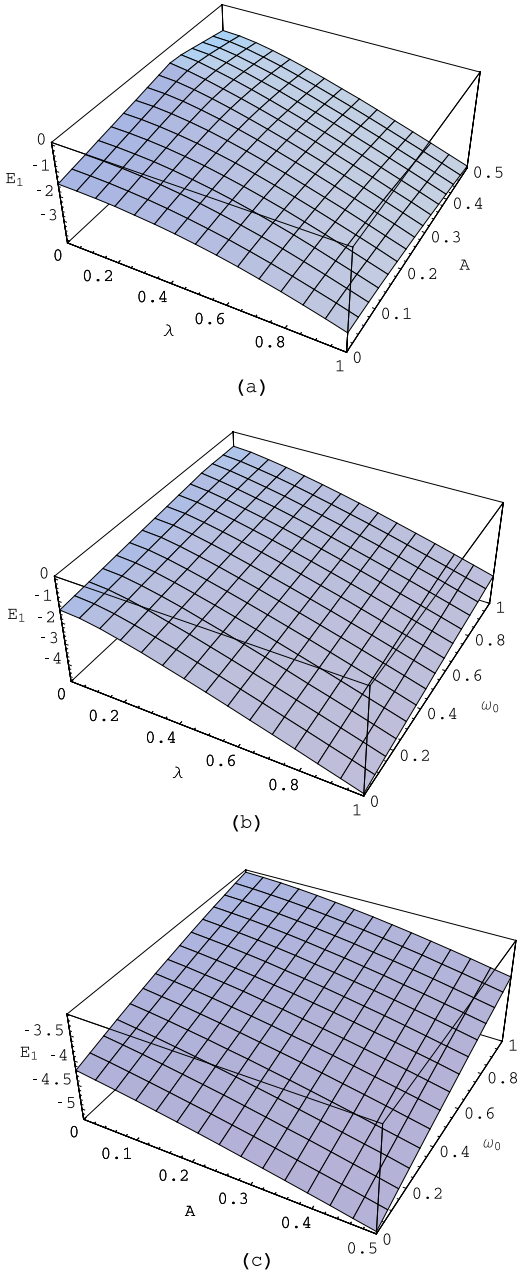

Figure 1. (a) The ground-state energy E_1 versus the coupling strength λ and the anisotropy constant A with $\omega_0 = \omega = 1$. (b) E_1 versus the coupling strength λ and the frequency ω_0 with $A = 0.3$ and $\omega = 1$. (c) E_1 versus the anisotropy constant A and the frequency ω_0 with $\lambda = 1$ and $\omega = 1$.

E_j are given by

$$|\psi_j\rangle = \frac{1}{N_j}\left(a_j\left|\frac{3}{2}, n\right\rangle + b_j\left|\frac{1}{2}, n+1\right\rangle + c_j\left|-\frac{1}{2}, n+2\right\rangle + d_j\left|-\frac{3}{2}, n+3\right\rangle\right), \tag{4}$$

where

$$a_j = \lambda\sqrt{3(n+1)}, \tag{5}$$

$$b_j = 3A - \frac{3\omega}{2} + n\omega_0 - E_j, \tag{6}$$

$$c_j = \frac{[-3A - \frac{\omega}{2} + (n+1)\omega_0 - E_j][3A - \frac{3\omega}{2} + n\omega_0 - E_j] - 3\lambda^2(n+1)}{2\lambda\sqrt{(n+2)}}, \tag{7}$$

$$d_j = \frac{\lambda\sqrt{3(n+3)}}{3A + \frac{3\omega}{2} + (n+3)\omega_0 - E_j} c_j, \tag{8}$$

$$N_j = \sqrt{|a_j|^2 + |b_j|^2 + |c_j|^2 + |d_j|^2}. \tag{9}$$

It is known that in the standard semiclassical framework the field operators a and $a^\dagger$ are replaced by the classical amplitude with rotation factors $e^{-i\varphi(t)}$ and $e^{i\varphi(t)}$ with $\varphi(t) = \omega_0 t$. Therefore, the semiclassical Hamiltonian corresponding to Hamiltonian (1) can be written as $H = A(3S_z^2 - S^2) - \Delta S_z - \lambda(\alpha S_+ e^{-i\varphi(t)} + \alpha e^{i\varphi(t)} S_-)$, where Δ is the detuning parameter and α is the amplitude of the oscillating field. It can be seen easily that this semiclassical Hamiltonian can be expressed in terms of an effective vector field $\mathbf{B} = (2\lambda\alpha\cos\varphi, 2\lambda\alpha\sin\varphi, \Delta)$ as $H = A(3S_z^2 - S^2) - \mathbf{B}\cdot\mathbf{S}$. When $\varphi(t) = \omega_0 t$ is changed adiabatically and periodically in the parameter space of the effective vector field $\mathbf{B}$, the semiclassical Berry phase can also be obtained [38]. However, if the controlled external field is quantized, this effective vector field becomes part of the system itself and therefore cannot be taken into account any longer as an external variable. But the corresponding state can also be manipulated in the parameter space of the coupling strength λ for Hamiltonian (1). Following the spirit of [10, 11], this Berry phase can be evaluated by introducing the following phase shift:

$$R(t) = \exp[-i\varphi(t)a^\dagger a], \tag{10}$$

where $\varphi(t) = \omega_0 t$ should be changed adiabatically and periodically.

This phase shift $R(t)$ can lead to the time-dependent transformation $|\Psi_j(t)\rangle = R(t)|\psi_j\rangle$ or $|\psi_j\rangle = R^\dagger(t)|\Psi_j(t)\rangle$, where $|\psi_j\rangle$ is the eigenvector of the time-independent eigenequation $H|\psi_j\rangle = E_j|\psi_j\rangle$ and $|\Psi_j(t)\rangle$ is the eigenvector of the time-dependent eigenequation $i\,d|\Psi_j(t)\rangle/dt = H'(t)|\Psi_j(t)\rangle$ with $H'(t) = R(t)HR^\dagger(t) - iR(t)\,dR^\dagger(t)/dt$. For the time-dependent eigenequation $i\,d|\Psi_j(t)\rangle/dt = H'(t)|\Psi_j(t)\rangle$ the Berry phase can be calculated in terms of standard definition using $\gamma_j = i\int_0^T \langle\Psi_j(t)|\frac{d}{dt}|\Psi_j(t)\rangle\,dt = i\int_0^{2\pi}\langle\Psi_j(\varphi)|\frac{d}{d\varphi}|\Psi_j(\varphi)\rangle\,d\varphi$. By using the transformations $|\Psi_j(t)\rangle = R(t)|\psi_j\rangle$ and $\langle\Psi_j(t)| = \langle\psi_j|R^\dagger(t)$, the final Berry phase corresponding to Hamiltonian (1) can be given as

$$\gamma_j = i\int_0^{2\pi} \left\langle\psi_j\right| R^\dagger(\varphi)\frac{d}{d\varphi}R(\varphi)\left|\psi_j\right\rangle d\varphi. \tag{11}$$

For the spin-1/2 Jaynes–Cummings model whose Hamiltonian reads $H^{JC} = \omega_0 a^\dagger a + \omega\sigma_z/2 + \lambda(\sigma_+ a + \sigma_- a^\dagger)$, the Berry phase can be evaluated as $\gamma_+ = \pi(1 - \cos\theta_n) + 2\pi n$ and $\gamma_- = -\pi(1 - \cos\theta_n) + 2\pi(n+1)$ with $\cos\theta_n = (\omega - \omega_0)/\sqrt{(\omega - \omega_0)^2 + 4\lambda^2(n+1)}$, which can be mapped into the semiclassical results in the coherent state representation with large amplitude [10]. However, for Hamiltonian (1) the Berry phase can be derived from equations (4)–(11) as the following:

$$\gamma_j = \frac{2\pi}{N_j^2}[n|a_j|^2 + (n+1)|b_j|^2 + (n+2)|c_j|^2 + (n+3)|d_j|^2]. \tag{12}$$

This is very interesting for discussions on the ground-state Berry phase ($n = 0$). A novel observation of this paper is that the ground-state Berry phase can be controlled by both the

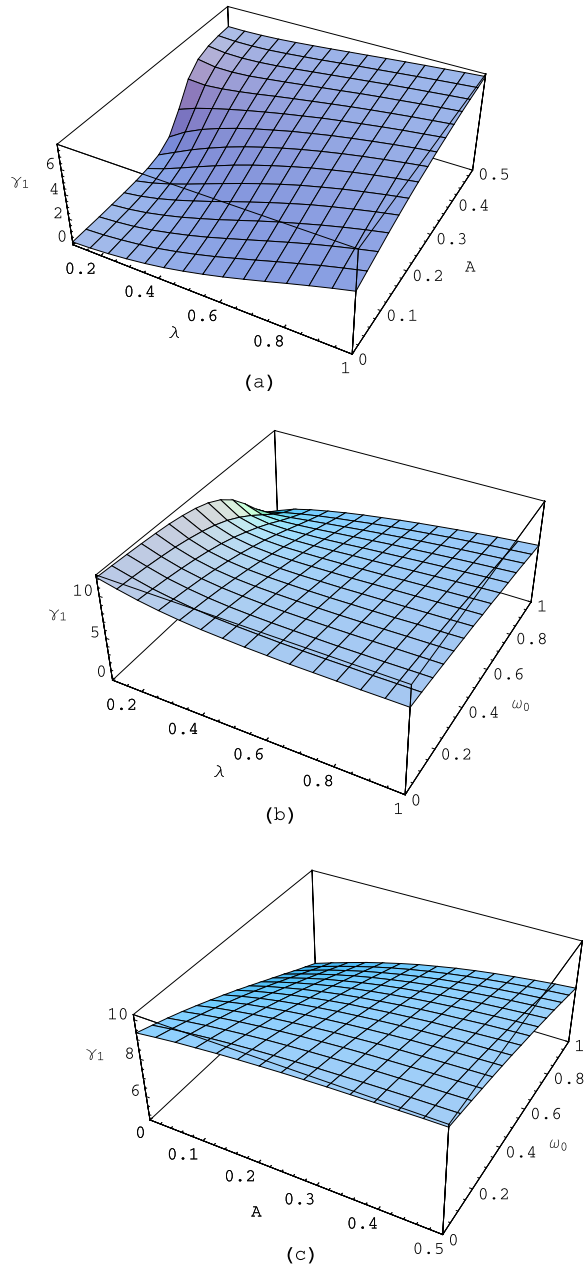

Figure 2. (a) The ground-state Berry phase γ_1 versus the coupling strength λ and the anisotropy constant A with $\omega_0 = \omega = 1$. (b) γ_1 versus the coupling strength λ and the frequency ω_0 with $A = 0.3$ and $\omega = 1$. (c) γ_1 versus the anisotropy constant A and the frequency ω_0 with $\lambda = 1$ and $\omega = 1$.

coupling strength λ and the anisotropy constant A shown in figure 2(a), with the resonance case. The behaviour of the ground-state Berry phase is similar to that of the ground-state von Neumann entropy as a measure of the entanglement between the nuclear spin and photon [37].

However, the maximum value of the ground-state von Neumann entropy S is given by $S_{\max} = 2$ whereas the maximum value of the ground-state Berry phase can reach $(\gamma_1)_{\max} = 4.6$. In fact, the ground-state Berry phase can also be easily controlled by adjusting the coupling strength λ and the frequency ω_0 of the external quantized field shown in figure 2(b). Figure 2(c) shows the ground-state Berry phase as a function of the anisotropy constant A and the frequency ω_0.

In conclusion, the Berry phase in GaAs semiconductor with a quantized field has been obtained analytically. A novel feature of the Berry phase can be controlled by the coupling strength λ, the anisotropy constant A and the frequency ω_0 of the electromagnetic wave, which has important application in geometric quantum computing.

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